UNIVERSITY OF ROME
TOR VERGATA

CONTINUUM MODELING IN BIOMECHANICS

Moussa Saad

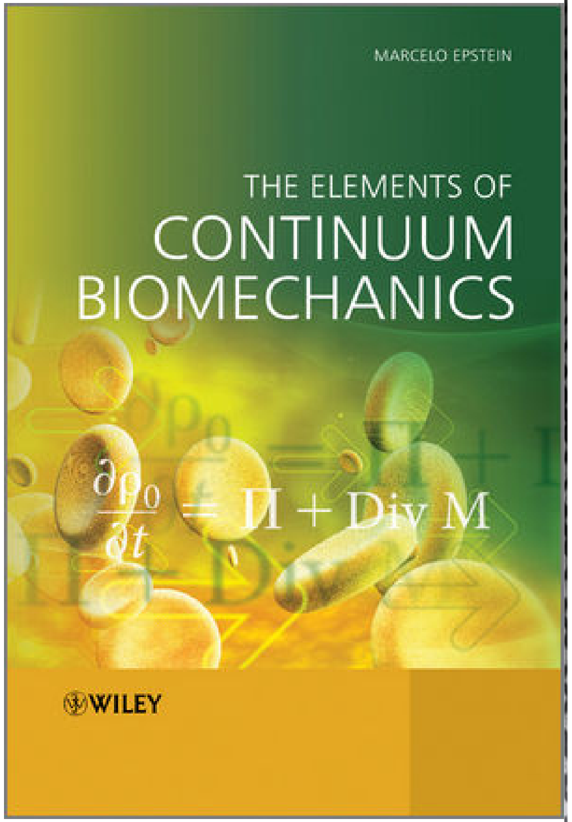
Advisors
Giuseppe Tomassetti
Andrea Micheletti

Main Reference:

THE ELEMENTS OF CONTINUUM BIOMECHANICS

Case Study: Blood flow as a traffic problem

Topics: Continuum approach
Motion description
Balance laws
Traffic flow behavior
Greenshields model
Linear traffic waves
Method of characteristics



Blood flow as a traffic problem

If we consider an artery with obstacles the situation can be compared to a highway populated by travelling vehicles where in this case those cars are cells of the blood.

The quantity to be balanced is the number of cells

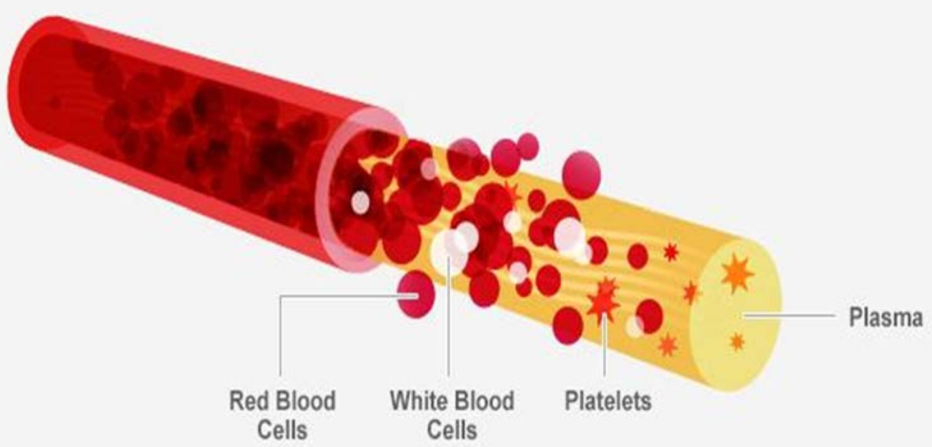
Blood

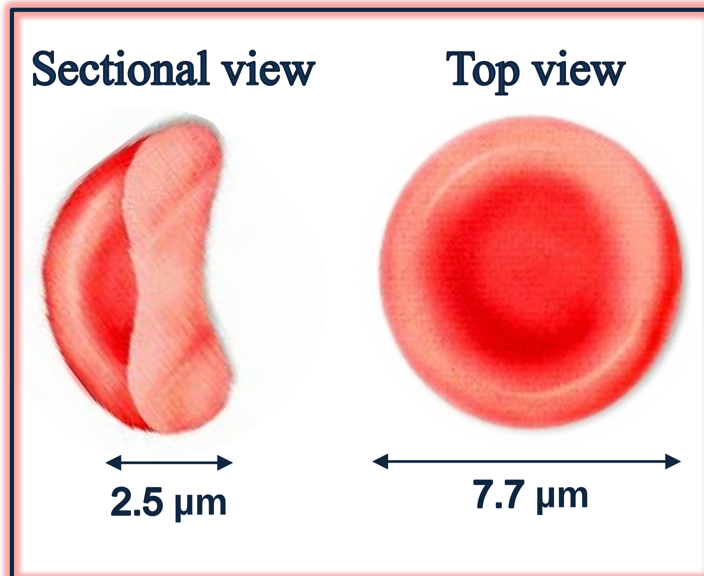
Plasma (54.3%)

Red blood cells (45%)

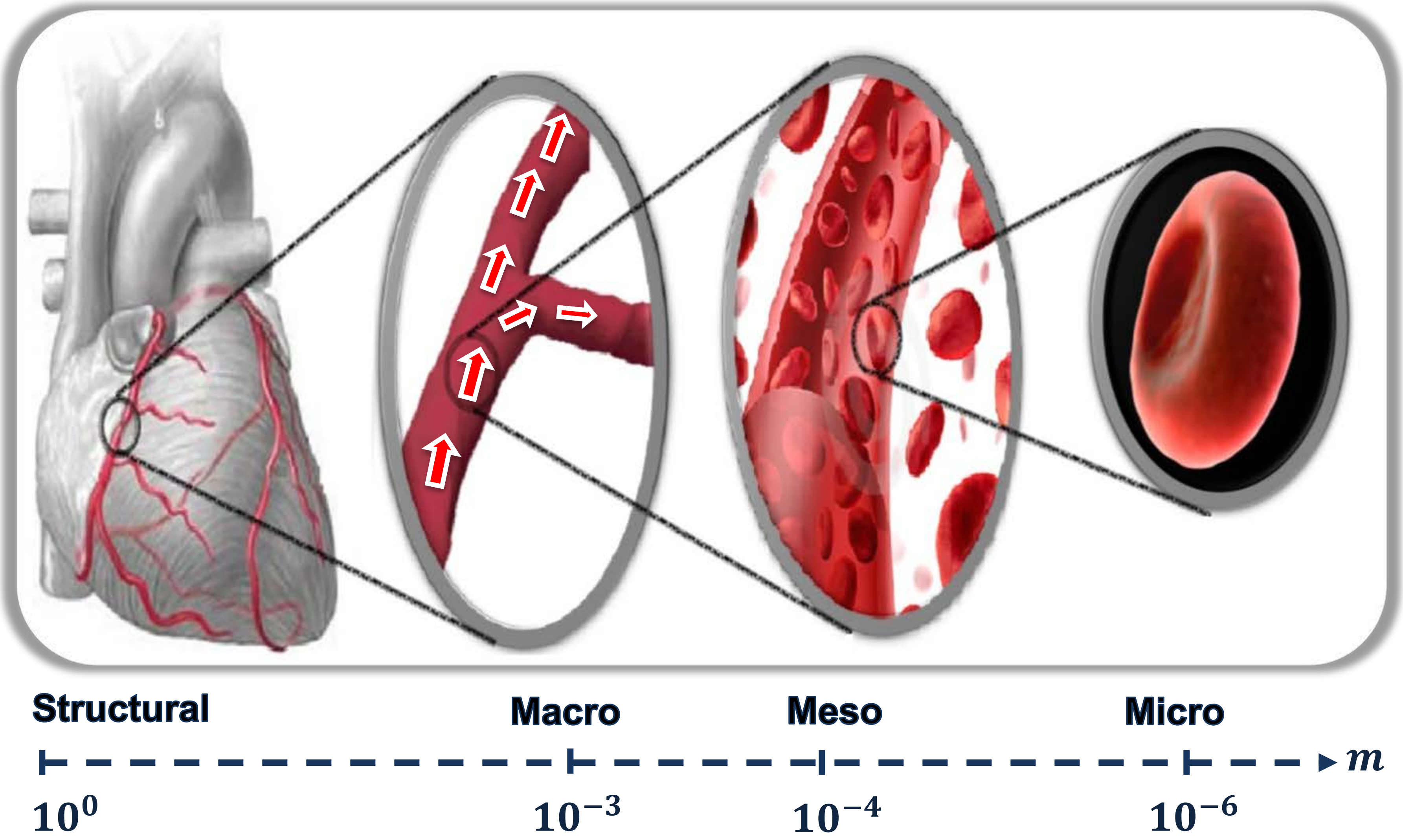
other cells (0.7%)

Blood vessel

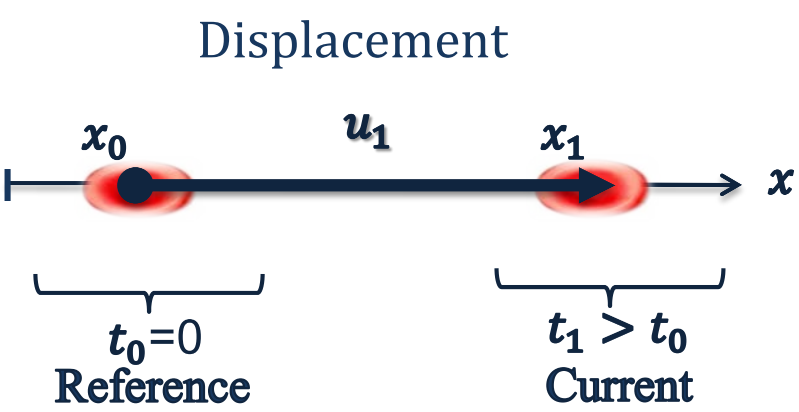


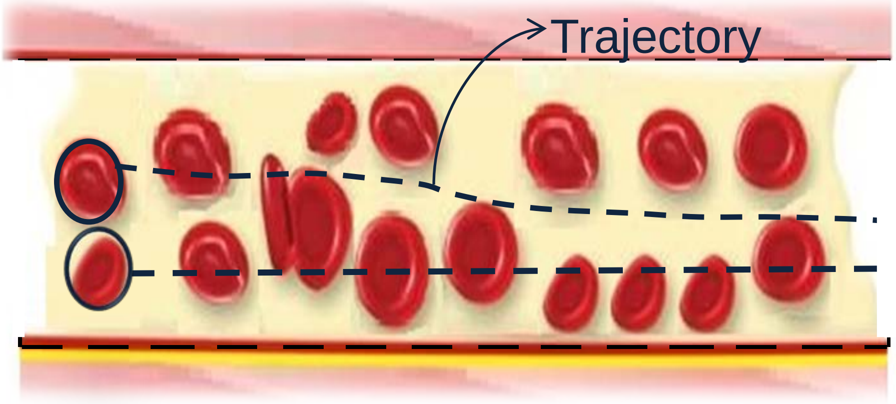


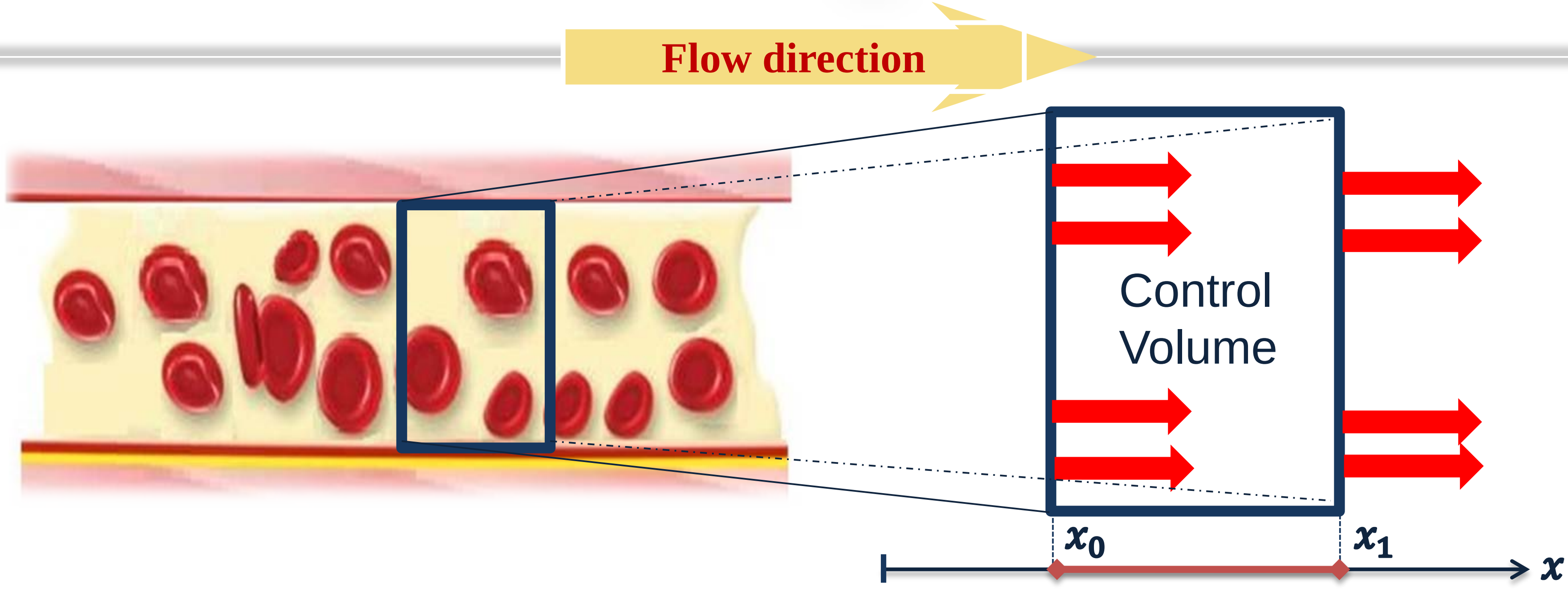
Continuum approach



Motion description







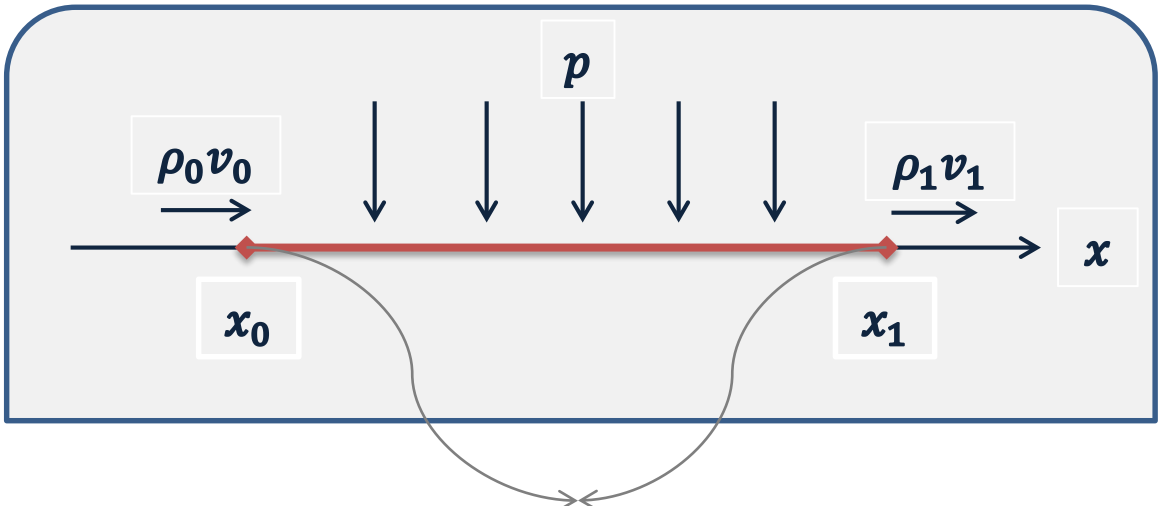
The structure of a balance law

$$\frac{d(content)}{dt} = P + F$$

P : Production term

F : Flux term

$$\frac{d(content)}{dt} = P + F$$



Boundary points

$$\frac{d}{dt}\int_{x_0}^{x_1} \rho(x,t)dx = \int_{x_0}^{x_1} p(x,t)dx + \rho(x_0,t)v(x_0,t) - \rho(x_1,t)v(x_1,t)$$

No diffusion

Only flux through fixed domain

1-D conservation equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0$$

Traffic flow behavior

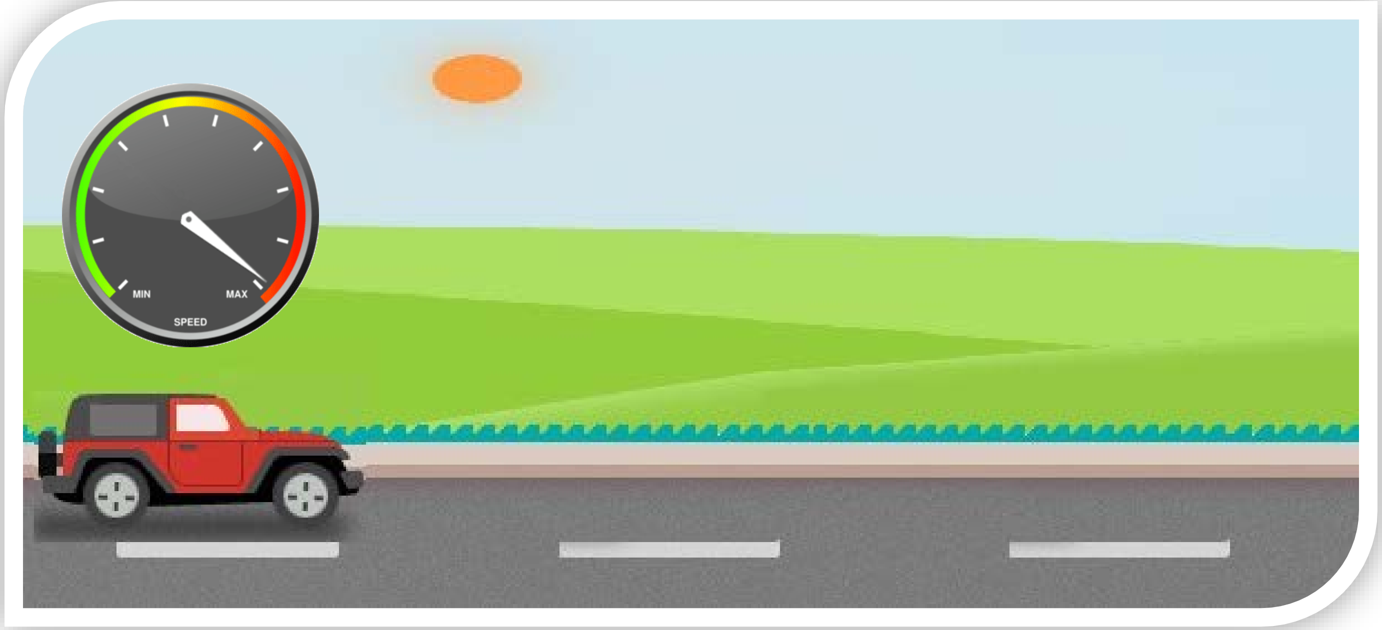
The flow
(Macroscopic)

Individual cars
(Microscopic)

The observed behavior of cars involves
- **driving speeds**
- **driving lane**

Factors

Traffic conditions
Cars characteristics
Road conditions
External agents

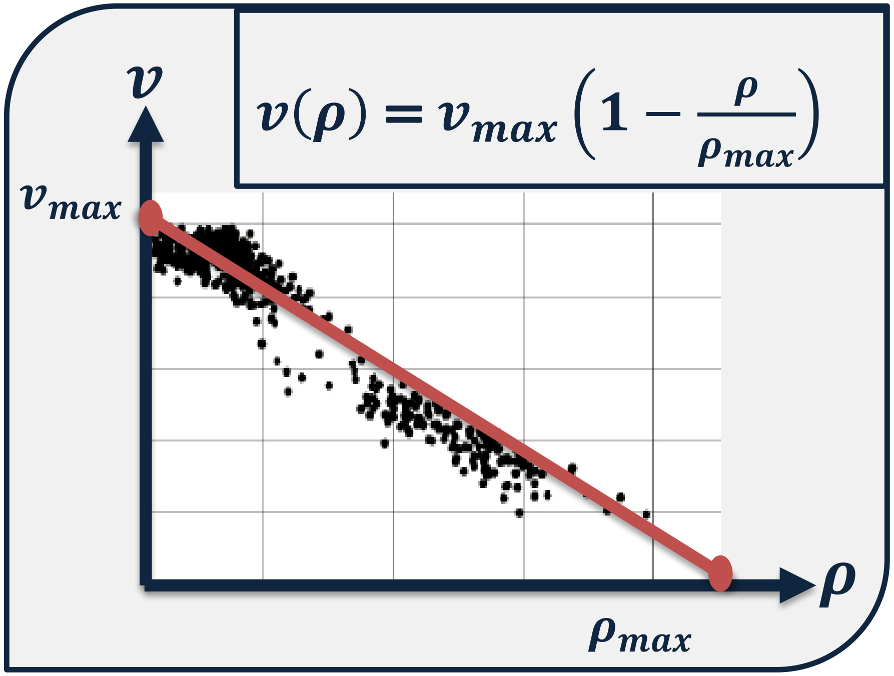

MIN
MAX
SPEED

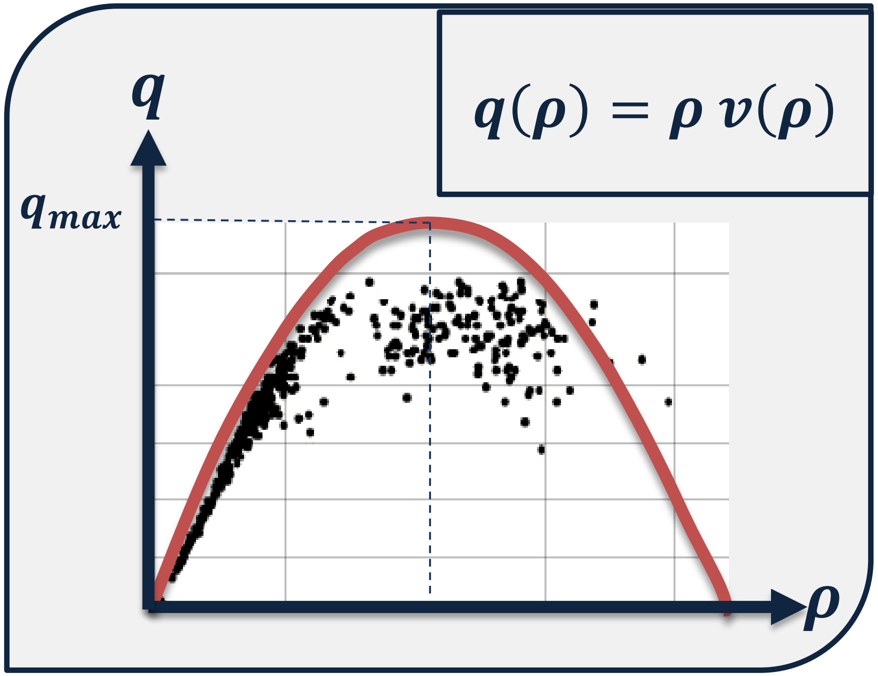

MIN
MAX
SPEED

Greenshields model

Constitutive law

$$v = f\,(\rho)$$





$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0$$

$$v(\rho) = v_{max}\left(1 - \frac{\rho}{\rho_{max}}\right)$$

$$q(\rho) = \rho\, v(\rho)$$

$$q(\rho) = v_{max}\left(\rho - \frac{\rho^2}{\rho_{max}}\right)$$

$$\frac{\partial \rho}{\partial t} + v_{max}\left(1 - \frac{2\rho}{\rho_{max}}\right)\frac{\partial \rho}{\partial x} = 0$$

$$q'(\rho) = v_{max}\left(1 - \frac{2\rho}{\rho_{max}}\right)$$

Quasi linear PDE

$$\frac{\partial \rho}{\partial t} + q'(\rho)\frac{\partial \rho}{\partial x} = 0$$

Linear traffic waves

$$\frac{\partial \rho}{\partial t} + q'(\rho)\frac{\partial \rho}{\partial x} = 0$$

$\rho = \rho_0 + \delta\rho$

$\delta\rho \ll \rho_0$

$$q'(\rho_0 + \delta\rho) = q'(\rho_0) + q''(\rho_0)\delta\rho + \ \ldots$$

Linearized form

$$\frac{\partial \rho}{\partial t} + q'(\rho_0)\frac{\partial \rho}{\partial x} = 0$$

$q'(\rho_0) = \ v_{max}\left(1 - \frac{2\rho_0}{\rho_{max}}\right) = c$

$$\frac{\partial \rho}{\partial t} + c\frac{\partial \rho}{\partial x} = 0$$

Solution form

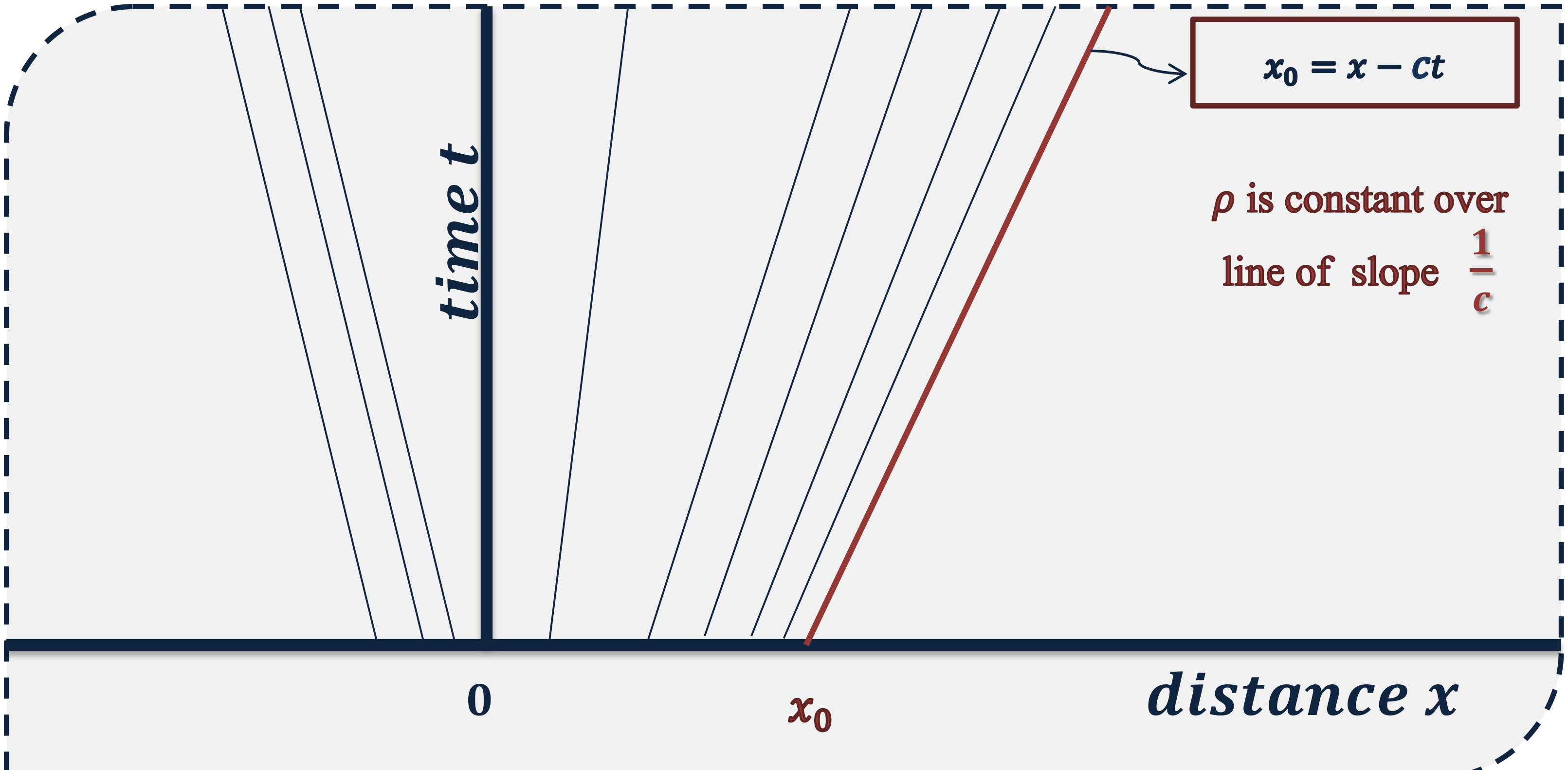
$$\rho = f\,(x - ct)$$

Method of characteristics

$$\frac{\partial \rho}{\partial t} + c\frac{\partial \rho}{\partial x} = 0$$

$$\frac{dx}{dt} = c$$

$$x = ct + x_0$$



Thank you for your attention